

# Decoupling Control for Underactuated ships' Trajectory Tracking

Yong Liu

(Navigation College, Dalian Maritime University, Dalian, China)

Email: [liuyong\\_dlm@163.com](mailto:liuyong_dlm@163.com)

Da-ming Sun and Hai-jun Xu

(Navigation College, Dalian Maritime University, Dalian, China)

**Abstract:**The tracking control problem is concerned for conventional surface ships with second order nonholonomic constraints. An output feedback algorithm was developed using decoupling control method and increment feedback based on iterative nonlinear sliding mode designing approach. With the known reference trajectory and measured position, the underactuated tracking control objective was achieved without the reference orientation to be generated by a ship model. The estimation of systemic uncertainty and disturbances and the yaw velocity PE (persistent excitation) conditions are not required. Computer simulation results on a full nonlinear hydrodynamic ship model of M.V. YULONG are provided to validate the effectiveness and robustness of the proposed controller in circle and sinusoidal reference tracking.

**Index Terms-**Underactuated ships; tracking control; decoupling control

## I. INTRODUCTION

The research of ship trajectory tracking control began from the 80<sup>th</sup> in 20<sup>th</sup> century<sup>[1]</sup>. The ship usually has underactuated characteristics<sup>[2]</sup> and nonholonomic constraints on the acceleration is non-integrable<sup>[3]</sup>, the system is not transformable into an equivalent system without drifts<sup>[4]</sup>. Recently, the significant research has paid much attention to this field<sup>[1-11]</sup>.

Trajectory tracking cannot be regulated to zero by coordinate transformation for the sake of rudder angle under drift caused by wind and current, and it must be compensated by a loxodrome (or sideslip compensation) since no sway control means are available. So, the equilibrium point of the system is not at the origin of transformed coordinates. Moreover, the only measurable state variables are the ship's position and heading in Earth fixed coordinates. Encarnacao<sup>[5]</sup> proposed a control method for estimation speed of stationary current based on feedback linearization and backstepping method as well as Aicardi<sup>[6]</sup> proposed the similar estimation. Breivik<sup>[7]</sup> proposed a method to control ship vector's direction instead of ship course which can only do the path following control. Han<sup>[8]</sup> used the differential flatness to design a control method and the method of direct dynamic feed-back linearization was adopted for the path tracking control of underactuated surface vessels.

The equivalence equations were derived and decoupled into two linear controllable systems. Then the control law was derived such that the tracking error with respect to the planned path could be stabilized asymptotically and globally, even under disturbance. DO based on Lyapunov's direct method and backstepping technique, propose a global nonlinear exponential observer to estimate unmeasured velocities<sup>[9]</sup>, and propose a methodology to design a controller that forces the position and orientation of underactuated ships, of which the sway axis is not actuated, and the mass and damping matrices are not assumed to be diagonal as often required in the literature, to globally track a reference trajectory<sup>[10]</sup>. The lasted achievement is a global controller that forces a ship without a sway actuator to follow a reference path and without velocity measurements for feedback. Nonlinear damping terms are also included to cover both low- and high-speed applications. Integral actions are added to the controller to compensate for a constant bias of environmental disturbances<sup>[11]</sup>. However, the method and design process are very complicated. Do<sup>[12]</sup> presents a constructive design of new controllers which based on a global exponential disturbance observer that force underactuated ships under constant or slow time-varying sea loads to asymptotically track a parameterized reference path. YU<sup>[13]</sup> proposed a new control law is similarly developed by introducing a first-order sliding surface in terms of surge tracking errors and a second-order surface in terms of lateral motion tracking errors which guarantee the convergence of position tracking errors. Most of the control methods above are depend on the accurate system model.

In this paper, considering the constraint conditions, external disturbances and uncertainties of ship motion control system, a robust nonlinear feedback decoupling control algorithm was proposed to control the vessel reach and follow a reference trajectory.

## II. PROBLEM STATEMENT

### *A The Kinematics and Dynamics Model*

The 3-DOF planar model of a surface vessel shown in Figure 1 is considered in this work. Considering the

trajectory tracking control problem of a surface vessel that has no side thruster, but one main thruster located at the center line in order to provide surge force. Neglecting the motions in heave, roll and pitch, the simplified kinematic model, which describes the geometrical relationship between the earth-fixed (E-frame) and the body fixed (B-frame) motion<sup>[14]</sup>, is given as

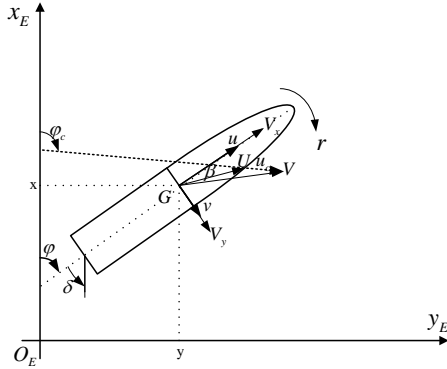


Fig.1: The earth-fixed and ship-fixed frames

$$\begin{cases} \dot{x} = u \cos \varphi - v \sin \varphi + u_c \cos \varphi_c \\ \dot{y} = u \sin \varphi + v \cos \varphi + u_c \sin \varphi_c \\ \dot{\varphi} = r \\ (m + m_x)\dot{u} - (m + m_y)vr = X_S + X_P + X_R + X_E \\ (m + m_y)\dot{v} + (m + m_x)ur = Y_S + Y_P + Y_R + Y_E \\ (I_{zz} + J_{zz})\dot{r} = N_S + N_P + N_R + N_E \end{cases} \quad (1)$$

where  $x$ ,  $y$  and  $\varphi$  are the longitudinal displacement, lateral displacement and heading angle, in the earth fixed frame,  $u$ ,  $v$  and  $r$  are longitudinal, lateral velocities over ground, and yaw angular rate in the ship-fixed frame,  $u_c$  and  $v_c$  denote longitudinal and lateral speed of current in the Earth fixed frame,  $\varphi_c$  is the direction of current. The vector summation of both determines the magnitude and direction of current.  $m$ ,  $m_x$ ,  $m_y$ ,  $I_{zz}$  and  $J_{zz}$  denote the ship inertia, added mass, and added moment of inertia.  $X$ ,  $Y$ ,  $N$  terms with subscripts  $H$ ,  $P$ ,  $R$ , and  $E$ , respectively, are longitudinal and lateral forces and moments induced by hydrodynamic damping, propeller, rudder, and other external effects except current

**B Assumption**

In practice, underactuated ship tracking control system is controllable when the external disturbances are bounded. According to characteristics of ship navigating at constant speed, do the following assumption:

**Assumption 1:** The control gain is bounded and sign is known. Define  $\frac{\partial N_R}{\partial \delta} > 0$ ,  $\frac{\partial X_P}{\partial n} > 0$  (Where  $\delta$  is rudder angle,  $n$  is main engine revolutions) ;

**Assumption 2:** The speed and acceleration and external disturbances are bounded and smooth while  $t \in \mathbb{R}^+$ .

The goal of trajectory tracking is to determine rudder angle  $\delta$  and main engine revolutions  $n$  and adjust speed to force the ship arrive at appointed position at prescribed

time.

**III. Trajectory-tracking controller design**

*A Calculation of track error*

Figure 2 is a reference coordinate frame, where  $(x_d, y_d)$  denote reference position,  $x_e$  denote longitudinal track error,  $y_e$  denote lateral track error and  $\varphi_e$  denote course deviation. If reference trajectory  $[x_d(t), y_d(t)]$  is known and smooth (Using waypoint interpolation method can determine smooth analytic equation<sup>[2]</sup>), while setting time  $t$  and ship position  $[x(t), y(t)]$ , the calculation of trajectory tracking error is following:

$$\begin{cases} \varphi_r = \text{atan2}[y'_d(t), x'_d(t)] \\ \theta_r = \text{atan2}[y(t) - y_d(t), x(t) - x_d(t)] \\ \rho_r = \{ [y(t) - y_d(t)]^2 + [x(t) - x_d(t)]^2 \}^{1/2} \\ x_e = \rho_r \cos(\theta_r - \varphi_r) \\ y_e = \rho_r \sin(\theta_r - \varphi_r) \\ \varphi_e = (\varphi - \varphi_r) \end{cases} \quad (2)$$

Where  $\varphi_r$  denote trajectory tangent direction of reference point,  $(\rho_r, \theta_r)$  denote ship position given in polar coordinate which original point is reference point,  $\text{atan2}(\cdot)$  denote arctan function in MATLAB (unit is rad) and range is  $[-\pi, \pi]$ .

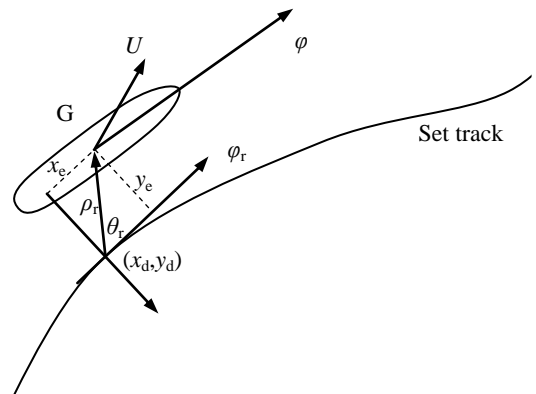


Fig. 2 Diagram of track error in trajectory tacking

In trajectory tracking control system, ignoring the error which caused by trajectory curvature and ship dimension as well as replacing the error in center of gravity by vessel reference point (VRP) is feasible.

*B Decoupling control based on nonlinear sliding mode method*

Regularly, ship course deviation  $\varphi_e < 90^\circ$ . It means ship could not track the trajectory in lateral and back direction. On this basis, a novel increment feedback algorithm which bases on nonlinear sliding mode is proposed for decoupling input-output variable of trajectory-tracking control system. The thrust force of propeller responsible for controlling the longitudinal track error  $x_e$  and the rudder responsible for controlling the lateral track error  $y_e$  and course deviation  $\varphi_e$  respectively.

The control law of longitudinal track error  $x_e$  is

designed as follows:

$$\begin{cases} \sigma_1^1(x_e) = k_1^1 \tanh(k_2^1 x_e) + \dot{x}_e \\ \sigma_2^1(\sigma_1^1) = k_3^1 \tanh(k_4^1 \sigma_1^1) + \dot{\sigma}_1^1 \\ \dot{n} = -k_5^1 \sigma_2^1 - k_6^1 \text{sgn}(\sigma_2^1) \end{cases} \quad (3)$$

Where  $k_1^1, k_2^1, k_3^1, k_4^1, k_5^1, k_6^1 \in \mathbb{R}^+$  are designed parameters.

The control law of lateral track error  $y_e$  and course deviation  $\varphi_e$  are designed as follows:

$$\begin{cases} \sigma_1^2(y_e) = k_1^2 \tanh(k_2^2 y_e) + \dot{y}_e \\ \sigma_2^2(\sigma_1^2, \varphi_e) = \varphi_e + k_2^2 \int \tanh(\sigma_1^2) dt \\ \sigma_3^2(\sigma_2^2) = k_3^2 \tanh(\sigma_2^2) + \dot{\sigma}_2^2 \\ \sigma_4^2(\sigma_3^2) = k_4^2 \tanh(\sigma_3^2) + \dot{\sigma}_3^2 \\ \dot{\delta} = -k_5^2 \sigma_4^2 - k_6^2 \text{sgn}(\sigma_4^2) \end{cases} \quad (4)$$

Where  $k_0^2, k_1^2, k_2^2, k_3^2, k_4^2, k_5^2, k_6^2 \in \mathbb{R}^+$  are designed parameters.

#### IV. STABILITY ANALYSIS OF CLOSED-LOOP SYSTEM

##### A Increment feedback control

Consider zero order single output system defined as:

$$y = f(x, u, t) \quad (5)$$

Where,  $y \in \mathbb{R}$  is output,  $x \in \mathbb{R}^n$  is external disturbances,  $n \in \mathbb{Z}^+, u \in [-u_{\max}, u_{\max}] \subset \mathbb{R}$  is input,  $u_{\max}$  is constraint of input,  $f(\cdot)$  is smooth when  $x \in \mathbb{R}^n, t \in \mathbb{R}^+, u \in \mathbb{R}$ . When  $x \in \mathbb{R}^n, t \in \mathbb{R}^+$ , exist  $u^*(t) \in [-u_{\max}, u_{\max}]$  to ensure  $y=0$ , defines  $\frac{\partial f}{\partial u} \cdot \dot{u}^* > 0$ .

**Theorem 1:** The control law  $\dot{u} = -k_p y - \varepsilon \cdot \text{sgn}(y)$  (where  $k_p, \varepsilon \in \mathbb{R}^+$ ) ensures the asymptotic stability of system (5).

The change rate of output is limited as well as the change rate of  $u^*(t_1)$ . When  $\varepsilon > |\dot{u}^*|_{\max}$ ,  $(\frac{\partial f}{\partial u} \cdot \dot{u}^*|_{\max} = \frac{df}{dt}|_{\max})$ ,  $|u(t) - u^*(t)|$  decrease continuous.

When  $t \rightarrow \infty, u(t) \rightarrow u^*(t)$ , obtained the asymptotic stability of system.

**Theorem 2:** If  $t \rightarrow \infty, u^*(t) \rightarrow u_c$  (where  $u_c$  is a constant), the control law  $\dot{u} = -k_p y$  (where  $k_p \in \mathbb{R}^+$ ) ensures the asymptotic stability of system(5).

**Proof:** If  $t=t_1, y(t_1)>0$ , according to the sign of control input gain, yields:

$$-u_{\max} \leq u^*(t_1) < u(t_1) \leq u_{\max} \quad (6)$$

According to formula (6) and system continuity,  $u$  decrease continuous, and  $u^*(t) \rightarrow u_c$ , if  $t \rightarrow \infty, u(t) \rightarrow u^*(t) \rightarrow u_c$ , yields  $y \rightarrow 0$ .

**Theorem 3** If  $u^*(t)$  is periodical variation, the control law  $\dot{u} = -k_p y$  (where  $k_p \in \mathbb{R}^+$ ) ensures the practical stability of system(5).

**Proof:** If  $t=t_1$  and  $y(t_1)>0$ , according to formula (6) and system continuity,  $u$  decrease continuous. Because  $u^*(t)$  is bounded and periodical variation, exist  $t=t_2, u(t_2)=u^*(t_2), y(t_2)=0$  and  $t=t_3, u(t_3)=u^*(t_3), y(t_3)=0$ . According to the conditions of making system smooth, there should be an extreme value in the zero point of output. So, the system is bounded.

##### B Stability of closed-loop tracking control system

Decompose  $\sigma 21$ , yields:

$$\begin{aligned} \sigma_2^1(\sigma_1^1) &= k_3^1 \tanh(k_4^1 \sigma_1^1) + k_1^1 \frac{d}{dt} \tanh(k_2^1 x_e) + \ddot{x}_e \\ &= k_3^1 \tanh(k_4^1 \sigma_1^1) + k_1^1 k_2^1 \dot{x}_e / \cosh^2(k_2^1 x_e) + \ddot{x}_e \end{aligned} \quad (7)$$

Consider the system model (1), and ignore these dynamic variables which are unrelated to main engine revolutions, yields:

$$\frac{\partial \sigma_2^1}{\partial n} = \frac{\partial \ddot{x}_e}{\partial n} = \frac{\partial}{\partial n} (\dot{u} \cos \varphi_e - \dot{v} \sin \varphi_e) \quad (8)$$

Regularly, the transverse force is ignored while comparing it with thrust force. Then, the formula (8) is similar to:

$$\frac{\partial \sigma_2^1}{\partial n} \approx \frac{\partial}{\partial n} (\dot{u} \cos \varphi_e) = \frac{\partial X_p}{\partial n} \cos \varphi_e / (m + m_x) \quad (9)$$

According to assumption (1), yields:

$$\frac{\partial \sigma_2^1}{\partial n} > 0 \quad (10)$$

The hyperbolic tangent function and hyperbolic cosine function have strict bound. According to formula (7), if reference trajectory is smooth enough and system is controllable, there should be have  $k_1^1, k_2^1, k_3^1, k_4^1 \in \mathbb{R}^+$  and  $n^*(t) \in [-n_{\max}, n_{\max}]$  to ensure  $\sigma_2^1=0$ .

According to the theorem 1, increment feedback law  $\dot{n} = -k_5^1 \sigma_2^1 - k_6^1 \text{sgn}(\sigma_2^1)$  ensures the asymptotic stability of  $\sigma_2^1$  as long as reference trajectory and external disturbances smooth enough. According to the theorem 3, if  $k_6^1=0$ , increment feedback law  $\dot{n} = -k_5^1 \sigma_2^1 - k_6^1 \text{sgn}(\sigma_2^1)$  ensures the practical stability of  $\sigma_2^1$  once external disturbances and curvature of trajectory variation periodical.

Decompose  $\sigma 4^2$ , yields:

$$\begin{aligned} \sigma_4^2(\sigma_3^2) &= k_4^2 \tanh(\sigma_3^2) + \\ &k_3^2 (r + \dot{\varphi}_r + k_2^2 \tanh(\sigma_1^2)) / (ch(\sigma_2^2))^2 + \\ &k_2^2 ((k_1^2 k_0^2 \dot{y}_e / (ch(k_0^2 y_e))^2 + \ddot{y}_e) / (ch(\sigma_1^2))^2 + \\ &(N_s + N_p + N_R + N_E) / (I_{zz} + J_{zz}) - \ddot{\varphi}_r \end{aligned} \quad (11)$$

Ignoring these dynamic variables which are unrelated to rudder angle, formula (11) can be rewritten:

$$\begin{aligned}
\frac{\partial \sigma_4^2}{\partial \delta} &= \frac{\partial}{\partial \delta} N_R / (I_{zz} + J_{zz}) + \frac{\partial}{\partial \delta} (k_2^2 \ddot{y}_e / (ch(\sigma_1^2))^2) \\
&= \frac{\partial}{\partial \delta} N_R / (I_{zz} + J_{zz}) + \\
&\quad k_2^2 \frac{\partial}{\partial \delta} (X_R \sin(\varphi_e) / (m + m_x)) / (ch(\sigma_1^2))^2 + \\
&\quad k_2^2 \frac{\partial}{\partial \delta} (Y_R \cos(\varphi_e) / (m + m_y)) / (ch(\sigma_1^2))^2
\end{aligned} \quad (12)$$

According to assumption 1 and bound of hyperbolic cosine, sine and cosine function, exists  $k_2^2$  to ensure:

$$\frac{\partial \sigma_4}{\partial \delta} > 0 \quad (13)$$

According to bound of hyperbolic function in formula(11) and assumption 2, exist  $\delta(t)^* \in [-\delta_{max}, \delta_{max}]$  to ensure  $\sigma_4^2=0$  (if  $k_1^2, k_2^2, k_3^2, k_4^2 \rightarrow 0$  does not exist, the system is non-controllable).

According to the definition of  $\sigma_4^2$ ,  $\sigma_3^2$  and  $\sigma_2^2$ , if  $\sigma_4^2 \rightarrow 0$ , yields  $\sigma_3^2 \rightarrow 0$  and  $\sigma_2^2 \rightarrow 0$ , yields:

$$\varphi_e \rightarrow -k_2 \int \tanh(\sigma_1^2) dt \quad (14)$$

and

$$\dot{\varphi}_e \rightarrow -k_2 \tanh(\sigma_1^2) \quad (15)$$

Consider formula (1) and the definition of  $\sigma_1^2$ , yields:

$$\begin{aligned}
\sigma_1^2 &= k_1 \tanh(k_0 y_e) + u \sin \varphi_e + \\
&\quad v \cos \varphi_e + u_c \sin(\varphi_e - \varphi_r)
\end{aligned} \quad (16)$$

According to formula (16), due to the strict bound of hyperbolic function and trigonometric function, exist  $\alpha \in (0, \pi/2)$  and  $\varphi_e^*(t) \in [-\alpha, +\alpha]$  to ensure  $\sigma_1=0$ . Then, if  $-\alpha \leq \varphi_e \leq +\alpha$ , obtain the following:

$$\frac{\partial \sigma_1^2}{\partial \varphi_e} = u \cos \varphi_e - v \sin \varphi_e > 0 \quad (17)$$

Considering formula (15) as the simple proportional feedback with saturation characteristic and  $v \cos \varphi_e + u_c \sin(\varphi_e - \varphi_r)$  as the external disturbances of zero order system  $\sigma_1^2$ . According theorem 2, the simple (proportional) increment feedback can ensure asymptotic stability of  $\sigma_1$  as long as external disturbances are stationary or attenuate and the curvature of trajectory tend towards stability such as tend towards straight line or circle trajectory. According to the definition of  $\sigma_1$ , yields:

$$\dot{y} \rightarrow -k_1 \tanh(k_0 y) \quad (18)$$

If the external disturbances  $v \cos \varphi_e + u_c \sin(\varphi_e - \varphi_r)$  are periodical variation, formula (14) and formula (15) can ensure track error and  $\sigma_1^2$  practical stability at the same time. Regularly, lateral velocity of ship may be caused by ship handling, wind and wave, especially there are high frequency external disturbances, the low pass filter should be designed to protect the steering gear and reduce the power dissipation [2]. In nature, the kinds of

current are most ocean current, wind-driven current and tide. The velocity of these current are slow. At the same time, the huge inertia of ship is similar to low pass filter which can filter these excessive influences of external disturbances to ensure the accuracy of tracking control.

## V SIMULATION AND RESULTS ANALYSIS

In order to verify the feasibility of control algorithm, use the Simulink of MATLAB design the ship "YULONG" trajectory tracking control program to carry out the computer simulation in different conditions and analyze the results.

Primary ship model parameters: Length over all 139.8 m, Displacement 14635 t, Molded breadth 20.8 m, Draft of full load 8 m, Block coefficient 0.681. Hydrodynamic model is MMG (Mathematical Modeling Group) model, main engine model and steering gear model are first order inertia model. Where main engine time constant  $T_M=4s$  and steering gear time constant  $T_E=2.5s$ .

Parameters of controller:  $k_1^1=1, k_2^1=0.01, k_3^1=0.1, k_4^1=1, k_5^1=1, k_6^1=0, k_0^2=0.02, k_1^2=1.5, k_2^2=0.02, k_3^2=0.02, k_4^2=0.02, k_5^2=100, k_6^2=0$ .

### A Trajectory of circle reference tracking

Circle trajectory is one of the most important form in the trajectory tracking control, where the center of circle is (0, 0), the radius is 500m and the tracking time is 1000s. During the tracking period, ship speed changed as sinusoid, where the periodic time is 1000s and amplitude is 0.5m/s.

Ship initial state:  $x=0, y=-550m, u=3m \cdot s^{-1}, \varphi=020^\circ$ , main engine revolutions  $n=120rpm$ . The external disturbances: Direction of stationary current is  $240^\circ$ , Velocity of current is 1kn, Direction of superimposed reversing current is  $240 \sim 060^\circ$ , Periodic time is 12h, Amplitude is 1kn, Phase is  $-\pi/3$ , Direction of stationary wind is  $090^\circ$ , Speed of wind changed as sinusoid where periodic time is 1min, amplitude is  $10m \cdot s^{-1}$  and average is  $10m \cdot s^{-1}$ .

Figure 3 is inputs and outputs curved line where broken line is designed trajectory. Figure 4 is plan trajectory where broken line is designed trajectory and mark "o" represents at prescribed time  $t=[0, 200, 400, 600, 800, 1000]$  (unit is s) ship will arrive at the appointed position.

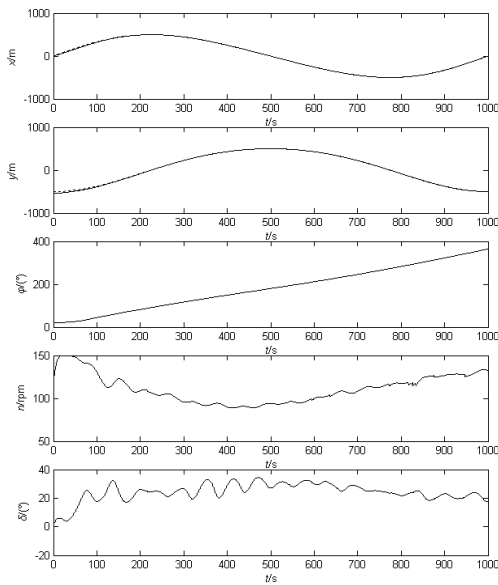


Fig.3 Inputs and outputs of circle reference tracking

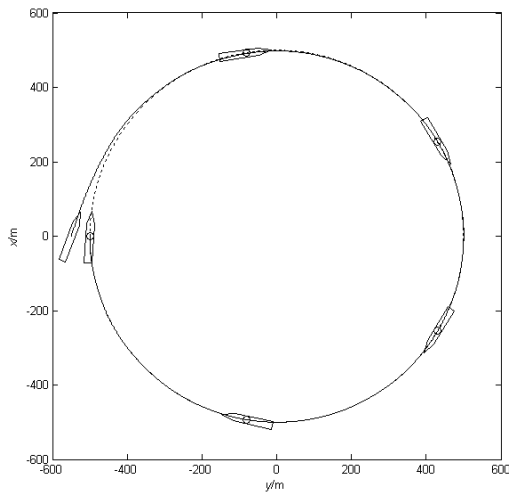


Fig. 4 Trajectory of circle reference tracking

**B Trajectory of sinusoidal reference tracking**

Sinusoidal trajectory is another important form in the trajectory tracking control, where designed trajectory is  $x=3t+\sin(\pi t/1000)$ ,  $y=200\sin(\pi x/1000)$ , amplitude is 200m, wave length is 2000m and the main purpose is to control ship arrive at appointed position at prescribed time.

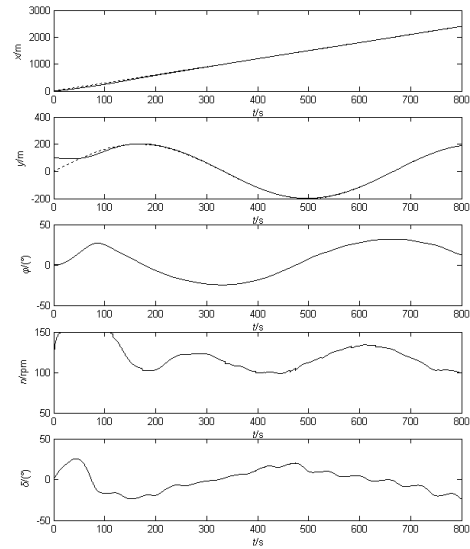


Fig.5 Inputs and outputs of sinusoidal reference tracking

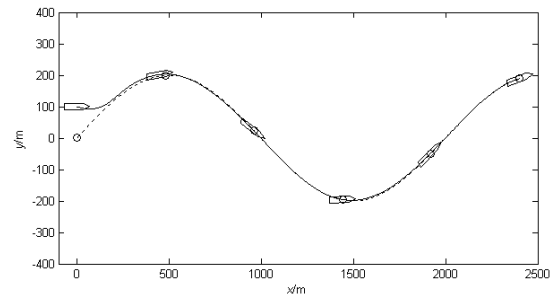


Fig. 6 Trajectory of sinusoidal reference tracking

Ship initial state:  $x=0, y=100m, u=2m \cdot s^{-1}, \varphi=000^\circ$ , main engine revolutions  $n=120rpm$ . External disturbances: water draft is 10m, direction of stationary current is  $210^\circ$ , velocity of current is 2kn, direction of superimposed reversing current is  $030\sim 210^\circ$ , periodic time is 12h, amplitude is 2kn, phase is  $-\pi/6$ , direction of stationary is  $150^\circ$ , Speed of wind changed as sinusoid where periodic time is 1min, amplitude is  $10m \cdot s^{-1}$  and average is  $10m \cdot s^{-1}$ .

By analyzing the results, summarize some conclusions as following:

- (1) Trajectory tracking error resulting from the external disturbances such as stationary current and wind have been overcome. And the trajectory tracing control is fast and smooth with lower power consumption.
- (2) The ship can be at the given reference position at the prescribed time shows that the accuracy of trajectory tracking control is very high.
- (3) The results indicate that trajectory tracking controller design method is valid and the trajectory tracking process conform navigation practice. The tracking controller achieves a good performance and has a powerful robustness to the uncertain system model, system state and external disturbance.

## VI Conclusions

Ship trajectory tracking control system has underactuated, nonholonomic constraints and lateral shift characteristics. In this paper, trajectory tracking output feedback controller was designed using increment feedback control based on nonlinear sliding mode method to decouple the trajectory tracking error. This output feedback controller does not need to calculate reference course and it can track the reference plan trajectory strict. The controller not depends on the accurate system model which can guarantee the convergence of tracking system although the external disturbances and ship velocity are not certain. The high precision ship-tracking controller is robust to the ship motion nonlinearity and the external disturbances.

## Acknowledgements

This work was supported by “Natural Science Foundation of Liaoning Province, China” (Number:201602084) and “The Fundamental Research Funds for the Central Universities”(Number : 3132016002,3132016006).

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**Yong Liu:** (1985-),Male, received the Master’s Degree in 2011 from Dalian Maritime University. From 2014, as a lecturer, he has been working in seaman ship office in Dalian Maritime University . His research fields include Ship Motion Control, Ship Handling, ship tracking control, ship path following control.

**Daming Sun:** (1982-),Male, received the Master’s Degree in 2007 from Dalian Maritime University. From 2010, as a lecturer, he has been working in seaman ship office in Dalian Maritime University . His research fields include Ship Management, Ship Handling, Ship Tracking Control, Ship Path Following Control.

**Haijun Xu:** (1984-),Male, received the Master’s Degree in 2010 from Dalian Maritime University. From 2013, as a lecturer, he has been working in seaman ship office in Dalian Maritime University . His research fields include Ship Motion Control, Ship Handling.